



Semester One Examination, 2019

Question/Answer booklet

**MATHEMATICS  
SPECIALIST  
UNIT 3**

**Section One:  
Calculator-free**

If required by your examination administrator, please place your student identification label in this box

Student number: In figures

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In words

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Your name

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**Time allowed for this section**

Reading time before commencing work: five minutes

Working time: fifty minutes

**Materials required/recommended for this section**

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	54	35
Section Two: Calculator-assumed	13	13	100	98	65
<b>Total</b>					100

## Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**Section One: Calculator-free**

**35% (54 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

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**Question 1**

**(5 marks)**

The equations of three planes are shown below.

$$\begin{aligned}x - y + 3z &= 11 \\x + 2y - 2z &= 0 \\x - y + z &= 9\end{aligned}$$

- (a) Determine the coordinates of the point of intersection of the planes. (3 marks)

- (b) Determine the distance of the point of intersection of the planes from the origin. (2 marks)

Question 2

(6 marks)

(a) Determine the modulus and argument of  $\frac{1}{1+i}$ .

(3 marks)

(b) Determine  $z^3$  in the form  $a + bi$ , where  $a, b \in \mathbb{R}$ , when  $z = 2 \cos\left(\frac{\pi}{18}\right) + 2i \sin\left(\frac{\pi}{18}\right)$ .  
(3 marks)

Question 3

(6 marks)

- (a) State whether the planes with equations  $2x + y - 3z = 3$  and  $4x + 3y + 4z = -3$  are perpendicular. Justify your answer. (2 marks)

- (b) Determine the Cartesian equation of the plane that passes through the three points with position vectors shown below. (4 marks)

$$\mathbf{a} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ 4 \\ -1 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Question 4

(6 marks)

Functions  $f$  and  $g$  are defined over their natural domains by  $f(x) = \sqrt{8-x}$  and  $g(x) = 3 + \frac{4}{\sqrt{x}}$ .

(a) State the domain of

(i)  $g(x)$ . (1 mark)

(ii)  $g^{-1}(x)$ . (2 marks)

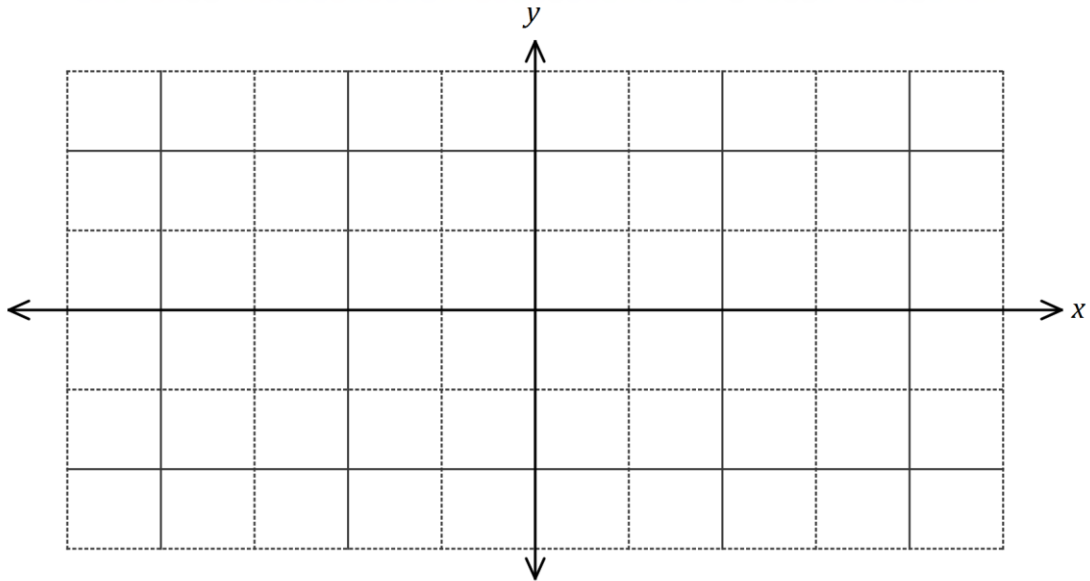
(b) Determine  $f \circ g(x)$  and the natural domain of this composite function. (3 marks)

Question 5

(7 marks)

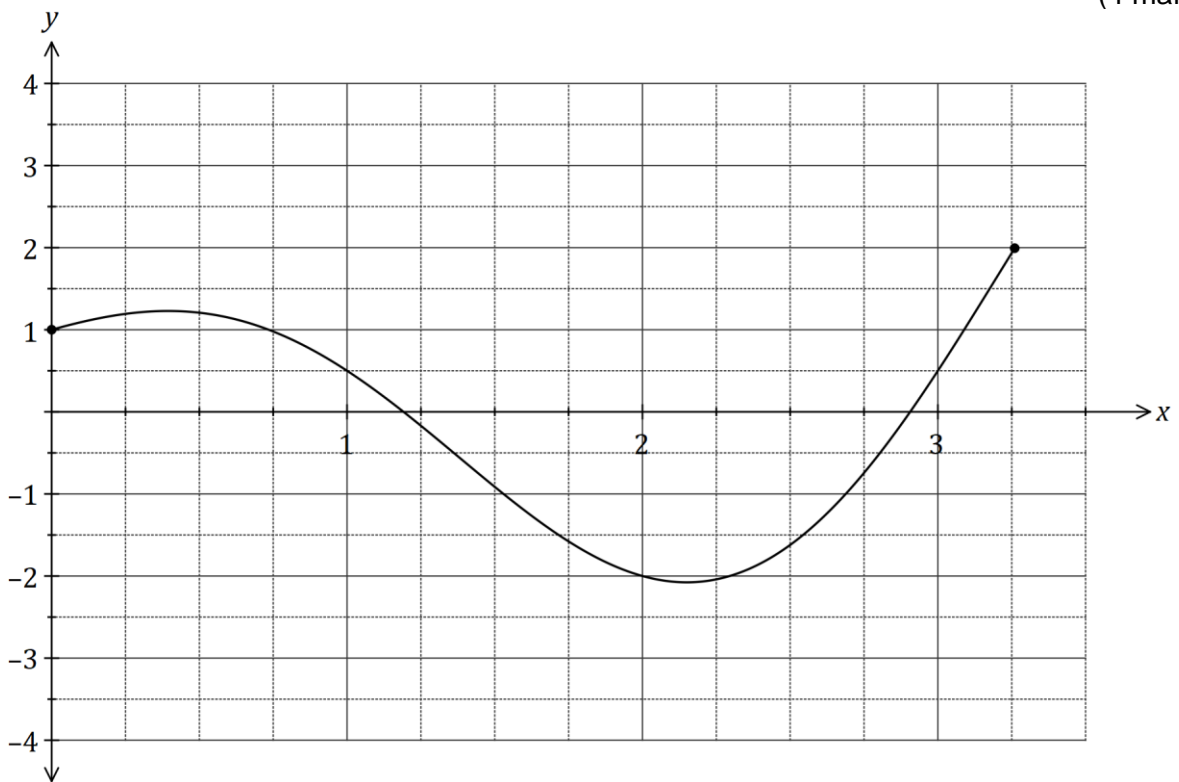
(a) On the axes below, sketch the graph of  $y = \frac{2x}{|x|}$ .

(3 marks)



(b) The graph of  $y = f(x)$  is shown below. On the same axes draw the graph of  $y = \frac{1}{f(x)}$ .

(4 marks)



**Question 6**

**(8 marks)**

Four functions are defined as

$$f(x) = 2x^2 - x - 1, \quad g(x) = x^2 - 2x - 3, \quad h(x) = x + 1, \quad k(x) = x - 3$$

Determine the equations of all asymptotes of the following graphs.

(a)  $y = \frac{h(x)}{g(x)}$ . (2 marks)

(b)  $y = \frac{g(x)}{f(x)}$ . (3 marks)

(c)  $y = \frac{f(x)}{k(x)}$ . (3 marks)



Question 7

(8 marks)

(a) Let  $z = x + yi$  and  $z^2 = p + qi$  where  $p, q, x, y \in \mathbb{R}$ .

(i) Determine an expression for  $p$  and for  $q$  in terms of  $x$  and  $y$ . (2 marks)

(ii) Show that  $\sqrt{p^2 + q^2} + p = 2x^2$ . (2 marks)

(b) (i) Solve the equation  $z^4 + 6z^2 + 25 = 0$  for  $z^2$ . (2 marks)

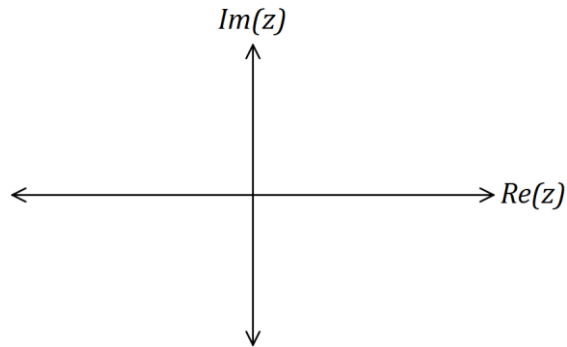
(ii) Using the result of (a), or otherwise, solve  $z^4 + 6z^2 + 25 = 0$ . (2 marks)

Question 8

(8 marks)

- (a) Sketch the locus of points  $z$  in the complex number determined by  $\arg(z + 3i) = \frac{3\pi}{4}$ .

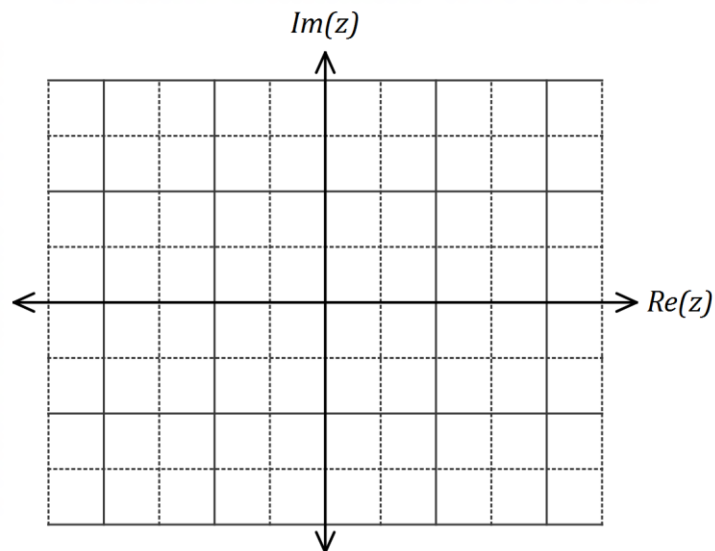
(3 marks)



- (b) Another locus of points  $z$  in the complex plane is determined by  $z\bar{z} + z + \bar{z} = 8$ .

- (i) Show that this locus can also be defined in the form  $|z - w| = k$ , clearly showing the value of constant  $w$  and the value of constant  $k$ . (3 marks)

- (ii) Sketch the locus on the axes below. (2 marks)



End of questions

Supplementary page

Question number: \_\_\_\_\_

